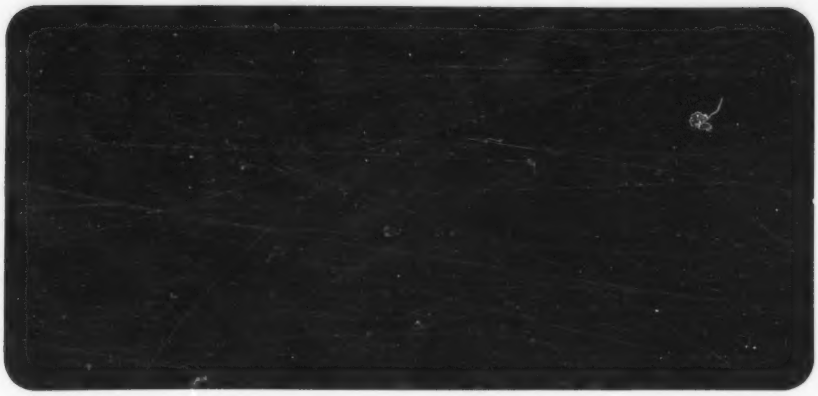
Working Paper



Department of Finance

Fiscal Policy and Economic Analysis Branch

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The Long-Run Economic Impacts of Government Debt Reduction

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Executive Summary

1. Introduction

This paper estimates the long-run effects of changes in Canadian government debt on key variables including real income, saving, investment, consumption, the capital stock and Canada's foreign indebtedness. The paper decomposes these into pure *savings channel* effects, that stem from the fact that lower deficits lead to higher total domestic saving, and *tax channel* effects that occur because lower debt servicing costs allow taxes to be reduced in the future. A dynamic computable general equilibrium model developed at the Department of Finance is used for the analysis.

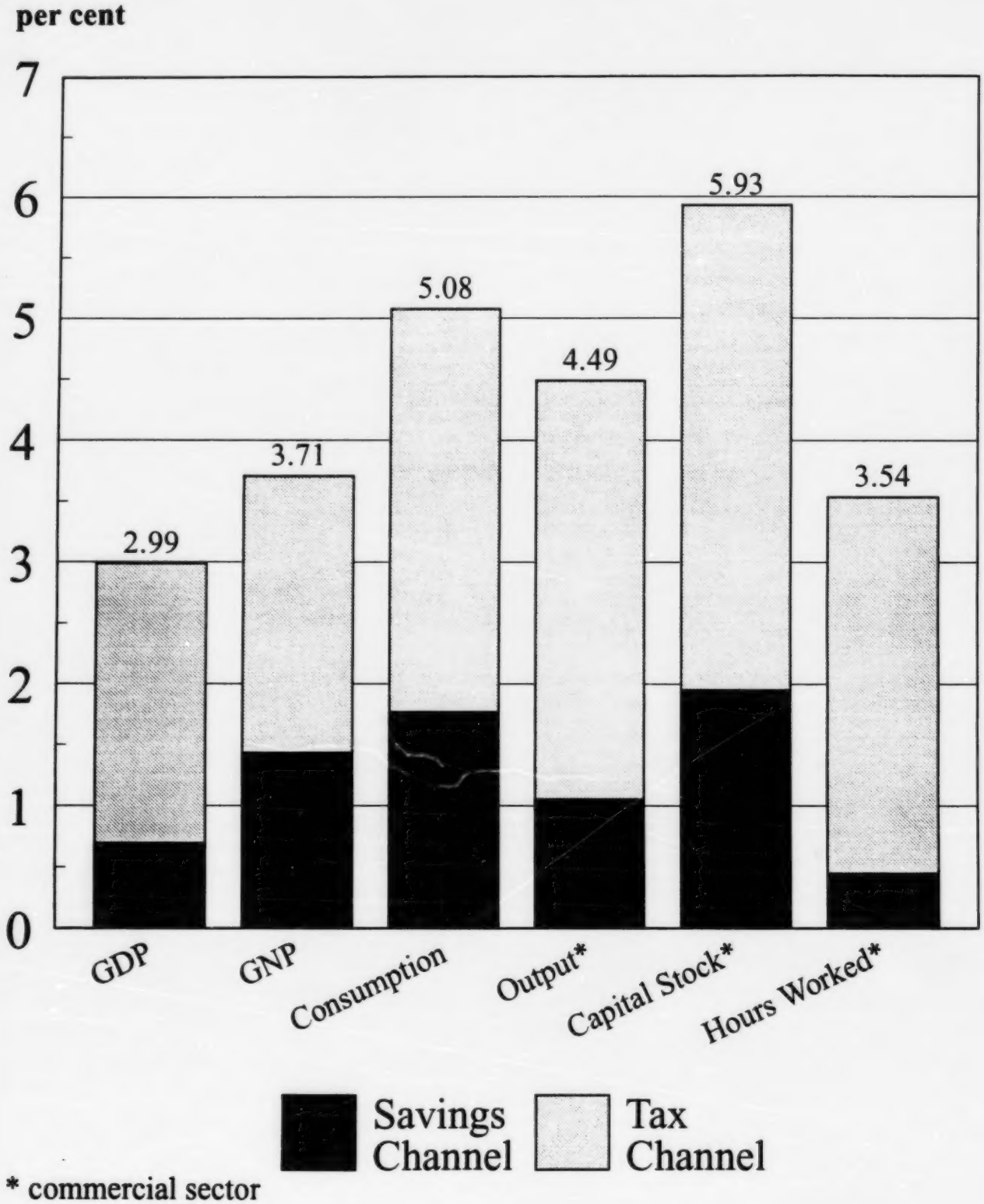
2. The Results

The experiment considered is a permanent reduction in the total government debt-to-GDP ratio of 25 percentage points. It is assumed that the resulting reduction in debt servicing costs is channelled into proportional reductions in income and payroll taxes, corporate and personal capital taxes, and indirect consumption taxes.

The simulation results are shown in Tables 1a and 1b and in Chart 1. They are decomposed, as described above, into impacts arising through increased national savings and those resulting from declining taxes.

The debt-to-GDP ratio reduction of 25 percentage points raises real commercial sector output in the long run by 4.5 per cent and real GDP by 3 per cent. Real GNP rises by more than GDP at 3.7 per cent because of the decrease in foreign indebtedness. Real consumption increases by 5.1 per cent, savings by 7.9 per cent, and hours worked by 3.5 per cent. The net foreign debt-to-GDP ratio falls by 7.2 percentage points. The tax reductions are responsible for more than half of the increases in real GDP, GNP, consumption, hours worked and the capital stock. The pass-through of higher public saving into total domestic saving is chiefly responsible for the fall in net foreign indebtedness. Public domestic saving rises by 0.75 per cent of GDP, with only one-fifth of this offset by a reduction in private domestic saving. Just under two-thirds of the

Chart 1: Long-Run Economic Impacts of a Debt-to-GDP Reduction of 25 Percentage Points (Base Case)



* commercial sector

increase in total domestic saving is channelled to higher gross investment, with the rest enabling a reduced reliance on foreign saving.

Table 1a: Long-Run Economic Impacts of Debt Reduction

(All real variables)

per cent shock minus control	Savings Channel	Tax Channel	**Total**
Gross Domestic Product	0.70	2.30	**2.99**
Gross National Product	1.43	2.28	**3.71**
Commercial Sector Output	1.04	3.44	**4.49**
Consumption	1.77	3.31	**5.08**
Total Domestic Saving	3.90	4.02	**7.92**
Capital Stock (Commercial Sector)	1.95	3.98	**5.93**
Hours Worked (Commercial Sector)	0.45	3.09	**3.54**

Table 1b: Long-Run Economic Impacts of Debt Reduction:

Impacts on Components of Savings and Net Foreign Debt

per cent of GDP: shock minus control	Savings Channel	Tax Channel	**Total**
Gross Savings / Gross Investment	0.16	0.22	**0.38**
Private Domestic Saving	-0.33	0.17	**-0.16**
Public Domestic Saving	0.72	0.03	**0.75**
Net Foreign Saving	-0.22	0.01	**-0.21**
Net Foreign Debt	-7.62	0.42	**-7.19**

The simulations have been conducted under the assumption that declines in the cost of government debt servicing are applied fully to uniform proportional reductions in all taxes, with government expenditures unchanged. The impacts would naturally be altered if the mix of declines in the range of taxes was different. The positive impacts would be larger the larger the declines in the most distorting taxes. They would, however, be smaller if expenditures were raised, or less distorting taxes were reduced instead.

A series of sensitivity tests demonstrate that the results are quite robust to a wide range of underlying assumptions used in the model.

Abstract

This paper seeks to determine the real long-run effects of reducing government debt in Canada. To perform the analysis, an open economy dynamic computable general equilibrium model is calibrated and simulated. The model's features include imperfect foreign-domestic asset substitutability resulting from home country asset preference, an overlapping generations framework that implies intergenerational disconnectedness and an endogenous labour-leisure decision that is affected by distorting taxes.

The model captures the real effects of government debt reduction through the failure of Ricardian equivalence to hold when generations are disconnected and through the reductions in distorting taxes that are made possible by stabilizing the debt-to-GDP ratio at a lower level. The paper estimates the relative importance of each channel and tests the robustness of the results to alternative parameterization assumptions.

The key result is that debt reduction, by increasing total saving and by allowing future distorting taxes to be reduced, leads to significant long-run output, income and consumption gains. Specifically, a debt-to-GDP ratio reduction of 25 percentage points is estimated in the long run to raise real commercial sector output by 4.5 per cent, real GDP by 3 per cent, real GNP by 3.7 per cent, and real consumption by 5.1 per cent. The net foreign debt-to-GDP ratio is estimated to fall by 7.2 percentage points. Sensitivity tests reveal that the quantitative results are quite robust to a wide range of alternative parameter assumptions.

Résumé

Dans la présente étude, les auteurs cherchent à déterminer les effets réels de long terme de la réduction de la dette du gouvernement canadien. L'analyse est effectué en étalonnant et en simulant un modèle dynamique d'équilibre général pour une économie ouverte. Le modèle incorpore des actifs domestiques et étrangers non parfaitement substituables provenant d'une préférence pour les actifs domestiques, des générations imbriqués modélisées telles que l'utilité de la génération présente ne prend pas en compte le bien-être des générations futures et un choix endogène travail-loisir influencé par la présence de taxe créant des distortions.

L'inclusion de taxes et l'absence d'altruisme intergénérationnel, empêchant ainsi l'équivalence Ricardienne, permet au modèle de capter les effets réels de la réduction de la dette. Les auteurs estiment l'importance relative des différents mécanismes de transmission et effectuent des tests pour évaluer la robustesse des résultats découlant d'hypothèses alternatives sur les paramètres du modèle.

Le résultat principal est que la réduction de la dette entraîne des gains à long terme pour la production, le revenu et la consommation par le biais d'une hausse de l'épargne agrégée et d'une baisse des taxes créant des distortions. En particulier, selon les estimations des auteurs, une réduction du ratio dette-PIB de 25 points de pourcentage augmente à long terme la production pour le secteur commercial de 4.5 pour cent, le PIB réel de 3 pour cent, le PNB réel de 3.7 pour cent et la consommation réelle de 5.1 pour cent. Le ratio de la dette extérieure sur le PIB diminue de 7.2 pour cent selon les estimations. Ces résultats sont robustes pour un ensemble assez grand d'hypothèses alternatives sur les paramètres du modèle.

1 Introduction

This paper estimates the real long-run effects of government debt reduction in Canada. The paper focuses on the pure savings channel effects arising from the failure of Ricardian Equivalence (described below) and on effects arising from reductions in distorting taxes made possible by lower debt. A dynamic computable general equilibrium model is used to conduct the analysis. The model's features include imperfect foreign-domestic asset substitutability resulting from home country asset preference, an overlapping generations framework that implies intergenerational disconnectedness and an endogenous labour-leisure decision that is affected by distorting taxes. James (1994) uses the same model to analyze the transition path effects of using various distorting taxes versus lump-sum taxes net of transfers to achieve the debt reduction. This paper, in contrast, focuses on the long-run benefits of debt reduction.

Section 2 reviews the relevant literature on the economics of debt reduction. Section 3 provides a detailed verbal description of the model and its calibration, with mathematical derivations given in an appendix. Section 4 provides the base case simulation results and conducts a series of sensitivity tests to determine the robustness of the results to alternative parameterization assumptions. Section 5 provides conclusions.

2 Review of the Literature

Issuing debt, or running a deficit, represents a delaying of taxes until the future. For this to have real economic effects, future taxes must be seen by agents as in some sense different from taxes today. If the future taxes associated with forgoing taxes today, or running a deficit, are seen as equal in present value to the current deficit, and if they are also non-distorting, or lump-sum, then no real economic effects follow. The notion that the debt-lump sum tax mix might be irrelevant is called "Ricardian equivalence" and has received renewed attention during the past fifteen years. Bernheim (1987) conducts an extensive literature review and argues that complete Ricardian Equivalence is theoretically highly improbable and is not supported empirically. Seater (1993) also reviews the literature, and argues that neither absolute nor approximate Ricardianism can be rejected. He notes that the theoretical objections to absolute Ricardianism suggest that it is nevertheless improbable. Absolute Ricardian equivalence would likely hold only under very special circumstances. For example, in a world where a representative intertemporal utility-maximizing agent faces an infinite horizon and has access to perfect capital markets, the debt-

lump-sum tax mix is irrelevant because government debt equals the present value of future primary surpluses. Private wealth equals the sum of private financial assets, human wealth (the present value of future labour income), government bonds and the negative of the present value of future primary surpluses. Government bonds make no net contribution to wealth and do not affect consumption. A government deficit raises disposable income and private saving by the amount of the deficit, however, total (private plus public) saving, the capital stock, output and net foreign indebtedness are unaffected by the path of public saving. Although people do not actually live forever, Ricardian equivalence may nevertheless hold. If parents regard their children as extensions of themselves as in Barro (1974), then the planning horizon of dynastic households is again infinite, and provided parents' bequest motives are operative as in Weil (1987), Ricardian equivalence prevails.

Ricardian equivalence likely fails for a number of reasons. Parents may be insufficiently altruistic regarding their children, some agents may have no children causing some household dynasties to be terminated as in Blanchard (1985), new households may appear that are disconnected from existing households as in Weil (1989) and the government may be capable of overcoming an adverse selection-based credit rationing problem generated by an inability of lenders to fully observe household risk characteristics as in Yotsuzuka (1987). If some degree of intergenerational disconnectedness exists, then the future taxes associated with a current deficit have a smaller present value than the new debt, hence government bonds are a net contribution to wealth. In a closed economy, a deficit today raises current wealth and consumption and lowers current investment leading to a long-run crowding-out of some private physical capital. Provided the economy is not dynamically inefficient, this lowers steady-state social welfare. In an open economy the consequences are a reduction in the consumption possibilities of future generations through a combination of increased net foreign indebtedness and a possibly lower capital stock. These impacts depend upon the degree of foreign-domestic asset substitutability and whether, as in Obstfeld (1989) and Macklem (1991), domestic untraded and relatively capital-intensive traded goods sectors exist. In such a case, given a high degree of asset substitutability, an increase in government debt leads to higher net foreign indebtedness. To maintain stability, a higher long-run trade surplus is required. This is achieved by increasing exports and, therefore, the capital stock of the traded goods sector.

For a given level of government expenditure, the level of debt may also have real consequences because of the use of distorting tax instruments to finance expenditures. This is conceptually separate from Ricardian equivalence *per se*, which asserts merely that the debt-lump-sum tax mix is irrelevant. Even if Ricardian equivalence holds, a reduction in the debt can allow future taxes to be lower, which will have real economic effects.

Theoretical general equilibrium models have been developed that provide an excellent framework for examining the qualitative implications of the debt-lump-sum tax mix. These are models that capture in various ways the intergenerational disconnectedness that underlies a possible failure of Ricardian equivalence. Blanchard's (1985) uncertain lifetimes approach that implies a constant positive probability of death, provides a powerful and tractable way of capturing intergenerational disconnectedness without the necessity of modelling discrete generational cohorts. This is achieved by specifying generations along a continuum. Government debt is a net contributor to wealth because future taxes are discounted by the probability of death plus the interest rate, implying that the present value of future primary balances is less than the stock of debt. Weil's (1985, 1989) introduction of population growth and new dynasties provides an additional source of disconnectedness. Even if existing households have infinite planning horizons, an incomplete altruism towards new future households causes Ricardian equivalence to fail. Buiter (1988) showed that a nonzero birth rate combined with the absence of operative intergenerational bequest motives yields debt nonneutrality in the Blanchard-Weil framework.

In this paper we quantify the long-run effects of debt reduction using a computable general equilibrium model embodying a plausible degree of non-Ricardianism, or intergenerational disconnectedness. Computable dynamic general equilibrium models with forward-looking model-consistent expectations have the advantage of being able to incorporate a degree of complexity and realism that would render a purely qualitative theoretical model analytically intractable. Recent examples include the overlapping generations life cycle model of Auerbach and Kotlikoff (1987), the model of Macklem (1991) based upon the theoretical framework of Weil (1985) and Obstfeld (1989), which is used to examine fiscal effects in a small open economy with a relatively capital intensive traded goods sector, and the models of James (1994) and Macklem, Tetlow and Rose (1994) .

Auerbach and Kotlikoff point out that the measured deficit or debt may be imperfect measures of the intergenerational consequences of government policies and that labels such as "debt" and "taxes" are often arbitrary and can mask the true nature of these consequences, leading to fiscal illusion. A progressive income tax, for example, potentially redistributes income from the current and future elderly, who have higher incomes, to the current young, while a pay-as-you-go pension scheme generates an unfunded liability of the government not captured in official debt measures. In both these cases important intergenerational redistributions may be occurring. The focus of this paper on the consequences of reducing "formal" debt should not be seen as a denial of the importance of these other sources of redistribution. The experiments should be seen more broadly as investigations of the possible effects of a change in any government-induced intergenerational redistribution, regardless of the labels involved. In any case, this paper is not concerned with measuring the effects of the level of redistribution which presently exists. This would be an ambitious task that would require an accurate measure of the intergenerational redistributions stemming from all government policies.

The model constructed reflects the dual focus on pure non-Ricardian effects arising from generational disconnectedness and effects arising from lower distorting taxes once the debt reduction has been achieved. Section 2 motivates and presents the model structure while section 3 provides and interprets the simulation results, compares them with those of other studies, and conducts a number of sensitivity tests. Section 4 provides a summary and conclusions. Mathematical derivations are provided in an annex.

3 The Model

3.1 Overview

The model combines the overlapping generations features of the Blanchard-Weil-Buiter uncertain lifetimes framework with the open economy asset substitutability, portfolio choice and capital tax treatment of Goulder and Eichengreen (1992). The former allows for the intergenerational disconnectedness necessary for incomplete Ricardianism, while the latter enables the effects of adjusting distorting capital taxes in a small open economy to be captured. The model also features endogenous labour supply, enabling the effects of changing distorting wage taxes to be determined.

3.2 Household Behaviour

The objective of all households is to maximize welfare subject to an intertemporal budget constraint or asset accumulation condition. Welfare consists of the sum of expected instantaneous utility flows discounted by a positive rate of time preference over an expected lifetime. If households die at a positive rate, this is equivalent to discounting over an infinite horizon at the rate of time preference plus the mortality rate. Expectations are model consistent.

A representative agent framework is most frequently employed in computable general equilibrium models. Such a framework has, however, the difficulty that it is unable to resolve the equity premium puzzle of Mehra and Prescott (1985). Mehra and Prescott found that over the period 1889-1979 the U.S. equity return averaged 7 per cent and the real 90 day treasury bill rate averaged 0.8 per cent, with a resulting average equity risk premium of 6.2 per cent. Using a representative agent model and empirically acceptable degrees of risk aversion, they were unable to reconcile so large a risk premium with so low a risk-free rate. One modelling solution is to simply ignore the problem and solve the representative agent's problem in a deterministic framework with the return on equity defined as the real after tax risk-free rate plus an exogenous premium, as is done in Goulder and Summers (1989). They justified this on the grounds that the true source of the premium is not well understood and may be determined by factors unrelated to risk.

Saito (1992) argued that the equity premium puzzle can be resolved in a heterogeneous agent framework. Motivated by the observation that the ownership of risky assets is confined to a relatively small group of agents he constructed a model in which a significant portion of consumers do not participate in the stock market owing to the existence of fixed participation costs. Aggregate risk is thus concentrated entirely on stockholders, who require a high compensation for bearing this risk. This paper's modelling of the household sector follows this heterogeneous agent approach. Two types of domestic and foreign households are assumed to exist: those that hold equity and corporate debt, and those that do not.

In order to ensure that the market value of an asset may not permanently grow faster than its rate of return, households are assumed to enforce a transversality condition upon private and government issuers of assets. In the case of bonds, this rules out Ponzi games (pyramid-type schemes) in which interest obligations can ultimately be met only through the issuance of yet

more bonds. In the case of equity, it excludes the possibility of eternal speculative bubbles in which share values exceed the levels warranted by income fundamentals. In practice this means that agents are assumed to be unwilling to hold an asset that is expected to be issued permanently at rate greater than its return. It is implemented by assuming that all asset stocks grow at the same rate as real GDP.

3.2.1 Corporate Asset-Holding Domestic Households

Corporate asset-holding domestic households choose paths for consumption of domestic and foreign goods and portfolio shares in order to maximize welfare subject to an asset accumulation condition. To simplify the portfolio choice problem, domestic households are assumed not to hold foreign government and corporate bonds. They receive no labour income, hence their wealth equals their financial asset holdings. Domestic and foreign corporate asset returns contain a stochastic component while government bonds are risk- free. Current utility is a function of consumption of the domestic and foreign goods and of the share of domestic assets in the household portfolio. The modelling of imperfect substitutability of foreign and domestic assets follows Goulder and Eichengreen (1992) and Kouri (1976) and reflects the observation that households seem to exhibit strong home country asset preference. Optimal consumption is proportional to wealth where the factor of proportionality is the rate of time preference plus the household mortality rate. The optimal portfolio shares are a function of expected rates of return and the variances and covariances of the various assets as in Merton (1971). The imperfect substitutability of foreign and domestic assets means that cross-country risk-adjusted return differentials can be consistent with diversified portfolios.

3.2.2 Non-Corporate Asset-Holding Domestic Households

Non-corporate asset-holding domestic households choose paths for consumption of the foreign and domestic goods, leisure and holdings of domestic government bonds in order to maximize welfare subject to an asset accumulation condition. They receive all domestic labour income, hence their wealth equals their financial asset holdings plus the discounted present value of future transfers net of wage, consumption and lump-sum taxes, plus human wealth, defined as the discounted present value of future time endowments. Future time endowments are discounted by the real after-personal tax return on domestic government bonds plus the household mortality rate. Current utility is a function of consumption of the domestic and

foreign goods and of leisure. The treatment of leisure follows King, Plosser and Rebelo (1987). Optimal consumption is proportional to wealth where the factor of proportionality is a function of current and expected future weighted differentials of the rate of time preference plus the household mortality rate and the real after-personal tax return on domestic government bonds. Optimal leisure is proportional to wealth expressed in units of time. Labour supply equals the time endowment minus consumption of leisure.

3.2.3 Foreign Households

Corporate asset-holding foreign households are modelled symmetrically to their domestic counterparts, except that they hold domestic corporate and government bonds as well as domestic equity and foreign assets. Non-corporate asset-holding foreign households are symmetric to their domestic counterparts.

3.3 Firm Behaviour

Representative firms choose levels of investment in order to maximize the market value of equity, which is the discounted value of after-tax dividends net of new share issues, subject to technology, physical capital accumulation and installation cost constraints.. The discount rate is determined by equating the required after-personal tax equity return with the sum of the after-tax dividend-price ratio and the after-tax rate of capital gain per share. The firm's technology is described by a constant returns to scale Cobb-Douglas production function. Trend total factor productivity and the time endowment evolve exogenously and physical capital depreciates at a constant rate. Installation costs are assumed to be a quadratic function of the rate of investment. Dividends paid to shareholders are the unretained portion of after-corporate tax earnings.

The approach to the solution of the investment problem follows that of Summers (1981, 1985), which synthesizes the q theory of investment developed by Tobin and the adjustment cost framework of Lucas (1967). The investment-capital ratio is a function of q, the percentage deviation of firm value from the replacement value of the capital stock, adjusted for capital consumption allowances and investment tax credits. The existence of installation costs which increase more than proportionally with the rate of investment causes firms to smooth their investment paths. Firms finance investment either by retaining earnings or issuing new debt. The investment decision generates the capital stock, and this combined with the labour input,

determines output via the production technology. The firm chooses the level of labour input in order to equate the marginal product of labour with the real wage, which it takes as given.

3.4 Government Behaviour

The government chooses paths for taxes, government debt and its expenditure mix. Government absorption is assumed to be equal to government output, thus the government's net contribution to society's consumption possibilities is zero. The stochastic portion of government capital tax revenue is assumed to be offset by the application of stochastic lump-sum capital taxes and transfers. This precludes an insurance capability for capital taxation described by Gordon (1985). Household enforcement of a transversality condition upon private and government issuers of assets means that the government may not permanently issue bonds at a rate faster than the real after tax return on government bonds. Steady-state stability of the government debt-to-GDP ratio ensures that this condition is satisfied. With a real after tax safe rate greater than the growth rate of output, a positive steady-state debt-to-GDP ratio requires steady-state primary surpluses.

3.5 General Equilibrium

The intratemporal equilibrium conditions are that given expectations, aggregate supplies and demands are equal at each point in time. Expectations must conform to the trend values realized in later periods. The goods, labour and foreign exchange markets clear, and firm financial structure is consistent with household portfolio choice.

Model consistent expectations of the trend values of variables are obtained using a variant of the Fair-Taylor extended path algorithm. The method is described in Fair and Taylor (1983).

3.6 Calibration

The steady-state version of the model is loosely calibrated to a 1991 Canadian benchmark. Table A.1 of the appendix gives the benchmark values of the principal calibration parameters. In the steady state, the trend income, expenditure and wealth variables grow at a rate of 3 per cent per year. This follows from the production function and the time endowment and trend total factor productivity growth rates, and implies a constant capital-labour ratio when labour is measured in efficiency units.

The calibration values of the elasticity of substitution between foreign and domestic assets, the momentary elasticity of labour supply and the household mortality rate are worthy of

particular attention. This is because the elasticity of substitution determines the openness of the economy to international financing flows, the momentary elasticity of labour supply determines the impact of an income-compensated real wage change on labour supply and the household mortality rate determines the degree of deviation from Ricardian Equivalence.

The base case elasticity of substitution between foreign and domestic assets is calibrated to be 1.4, which is close to the benchmark estimate of 1 used by Goulder and Eichengreen (1988) in their analysis of the general equilibrium effects of saving and investment-promoting tax policies.

The base case compensated wage elasticity of labour supply is assumed to be 1, based on a ratio of work to available time of 0.5. A smaller elasticity would naturally imply smaller wage tax effects, however this is already less than values typically calibrated in real business cycle models where the ratio of work to available time is assumed to be as low as .2 (see, for example, Prescott (1986) and Greenwood and Hercowitz (1991)) which implies a labour supply elasticity of 4. Microeconometric studies typically find smaller elasticities. Dahlby (1994), for example, assumes compensated labour supply elasticities of 0.2 to 0.4 in his examination of the marginal cost of public funds.

The base case household mortality rate is assumed to be 0.04, which implies a new household birthrate of 0.054, given a population growth rate of 0.014. This is consistent with an average expected remaining life of 25 years for adults. In theory, however, there is no strict link between observed individual mortality rates and the notional household mortality rates. A zero household birthrate could be consistent with positive individual birth and mortality rates. Our household mortality rate is, however, close to those of many studies that use the Blanchard-Buiter-Weil approach. For example, Macklem, Rose and Tetlow (1994) assume a household birthrate of 0.0532.

Owing to the importance of these key parameters, we subsequently conduct sensitivity tests in which we resimulate the effects of debt reduction using a wide range of alternative values.

4 Simulation Results

In all the scenarios considered the total government debt-to-GDP ratio is reduced from an initial steady-state national accounts value of .75 to a new steady-state value of .5, representing a

reduction of 33 per cent below the benchmark path. Wage taxes, source and personal capital taxes and indirect consumption taxes are all reduced by the same proportion in the long run. The tax reduction is endogenously determined by the smaller primary surplus that is consistent with the lower steady-state debt-to-GDP ratio.

The savings channel (SC) component of the long-run impact is determined by resimulating the debt reduction under the assumption that the tax reduction is lump sum. This allows the pure fiscal stance, or savings channel, effect to be isolated. If absolute Ricardian Equivalence held, then the savings channel impact would always be zero. The tax channel (TC) component represents the effect of reducing all distorting taxes proportionally, excluding the savings channel impact. This component would be nonzero even if absolute Ricardian Equivalence held.

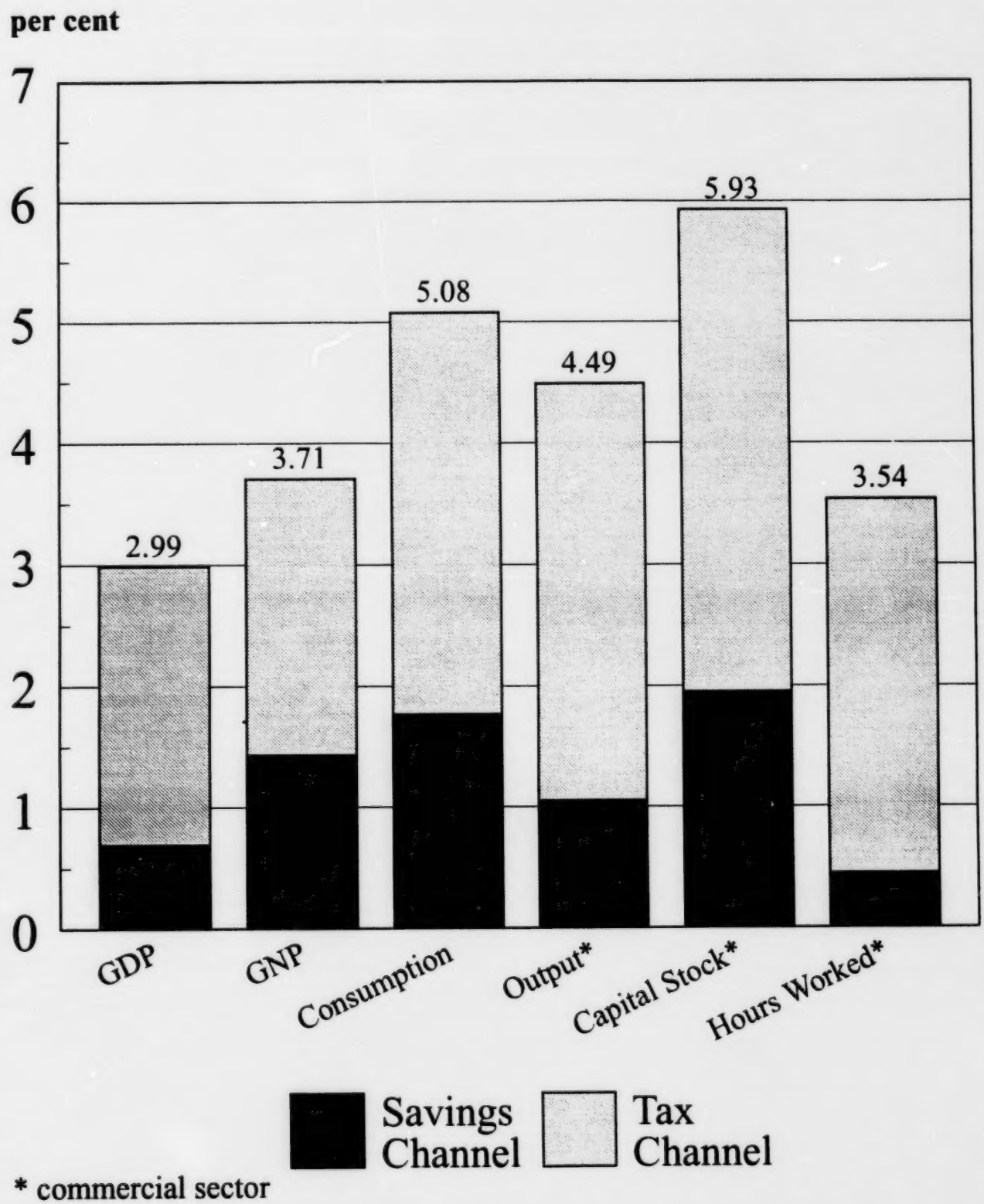
4.1 Base Case

The base case simulation results are shown in Tables 1a and 1b. The SC impact represents the effect of increased public saving *per se*, while the TC impact stems from the proportional decrease in all taxes that is made possible by the debt reduction.

Table 1a: Long-Run Economic Impacts of Debt Reduction (Base Case)

per cent shock minus control	Savings Channel	Tax Channel	**Total**
Gross Domestic Product	0.70	2.30	**2.99**
Gross National Product	1.43	2.28	**3.71**
Commercial Sector Output	1.04	3.44	**4.49**
Consumption	1.77	3.31	**5.08**
Total Domestic Saving	3.90	4.02	**7.92**
Capital Stock (Commercial Sector)	1.95	3.98	**5.93**
Hours Worked (Commercial Sector)	0.45	3.09	**3.54**

Chart 1a: Long-Run Economic Impacts of a Debt-to-GDP Reduction of 25 Percentage Points (Base Case)



* commercial sector

Table 1b: Long-Run Economic Impacts of Debt Reduction:

Impacts on Components of Savings and Net Foreign Debt (Base Case)

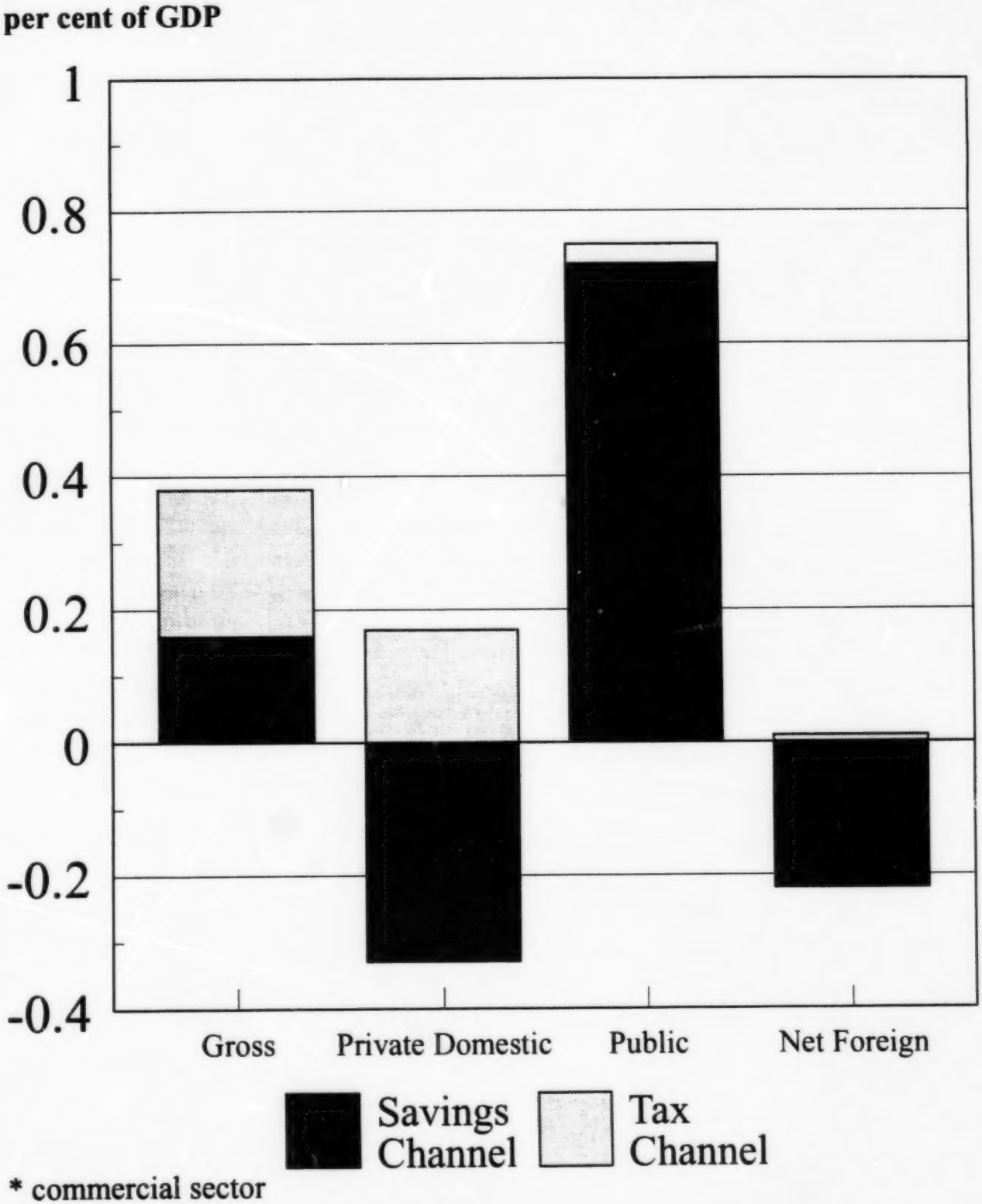
per cent of GDP: shock minus control	Savings Channel	Tax Channel	**Total**
Gross Savings / Gross Investment	0.16	0.22	**0.38**
Private Domestic Saving	-0.33	0.17	**-0.16**
Public Domestic Saving	0.72	0.03	**0.75**
Net Foreign Saving	-0.22	0.01	**-0.21**
Net Foreign Debt	-7.62	0.42	**-7.19**

The debt-to-GDP ratio reduction of 25 percentage points raises real GDP in the long-run by 3 per cent, real GNP by 3.7 per cent and real consumption by 5.1 per cent. The net foreign debt-to-GDP ratio falls by 7.2 percentage points. The TC effects form the main components of the GDP, GNP, consumption, hours worked and capital stock gains. The SC effect is the key determinant of the fall in net foreign indebtedness. The flow of public domestic saving rises by 0.75 per cent of GDP, and only one-fifth of this is offset by a reduction in private domestic saving. The SC offset is higher (about 45 per cent), however the TC effect actually boosts private domestic saving, yielding a total offset of 20 per cent.

The SC increase in total domestic saving raises the capital stock and reduces net foreign indebtedness. The higher capital stock in turn boosts labour productivity, which raises the demand for labour. The reduction in net foreign indebtedness generates an SC income gain double that of the SC GDP gain. The SC consumption gain is virtually the same as that found by Macklem, Rose and Tetlow (1994) given an equivalent debt reduction.

Reduced wage and consumption taxes are principally responsible for the TC increase in commercial sector hours worked.

Chart 1b: Long-Run Savings Component Impacts of a Debt-to-GDP Reduction of 25 Percentage Points (Base Case)



* commercial sector

4.2 Sensitivity Tests

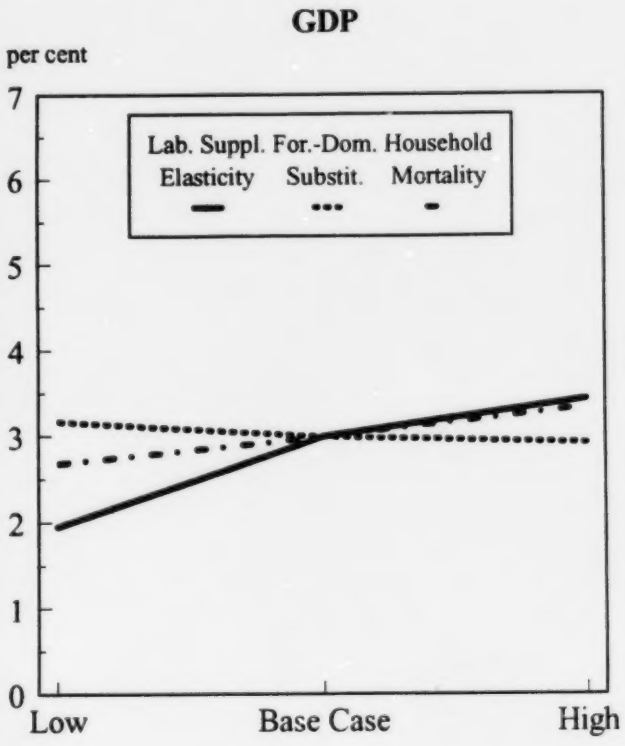
Table 2 provides the alternative parameter values used in conducting the sensitivity tests. The three key parameters previously described were varied: the elasticity of substitution between foreign and domestic assets, the momentary elasticity of labour supply and the household mortality rate.

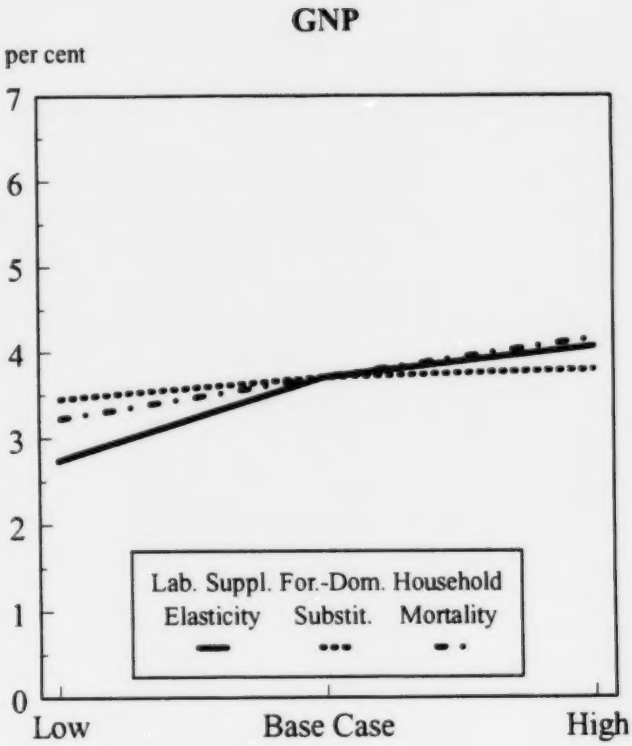
Table 2 Sensitivity Tests:
Base Case and Alternative Values

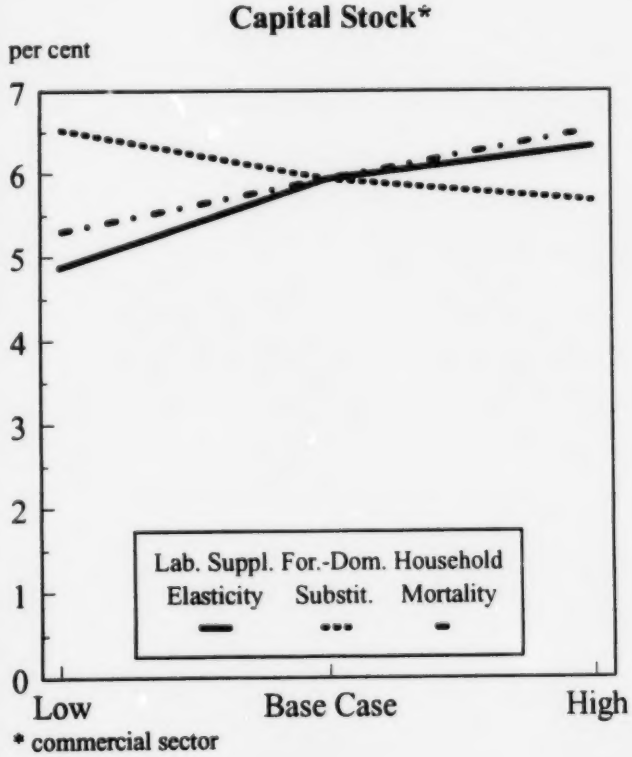
	Elasticity of Substitution Between Foreign and Domestic Assets	Elasticity of Labour Supply	Household Mortality Rate
Base Case Value	1.4	1.0	0.04
Alternative Value(s)	0.2 and 2.5	0.25 and 2.0	0.02 and 0.06

Tables 3a and 3b show the effects of changing the assumed momentary elasticity of labour supply from its benchmark value of 1 to 0.25 and 2.0, respectively. A lower labour supply elasticity increases the SC effect of the debt reduction on the capital stock. This follows because the lower elasticity is achieved by reducing the weight of leisure in the consumption bundle appearing in the non-corporate asset-holding household's utility function. This reduces the contribution of the present value of future time endowments to wealth, and therefore increases the share of a given change in the net debt contribution to total wealth, that is, the fiscal stance. The larger fiscal stance effect causes initial consumption to fall more, setting in motion a larger accumulation of physical capital and net foreign asset holdings and larger impacts on output and income. In contrast, a higher labour supply elasticity increases the TC effect of debt reduction on hours worked, and subsequently, on all other key variables. In long run general equilibrium the hours worked impact has a logarithmic relationship to the elasticity, with a doubling in the elasticity raising the impact by only 30 per cent. This is because in the long run a higher elasticity yields larger wealth gains which induces an increase in the consumption of leisure and a decrease in labour supply that partly offsets the substitution effect caused by the wage tax reduction. It is notable that assuming a labour supply elasticity of 0.25 still yields a commercial sector hours worked impact of 1.65 per cent, half that obtained in the base case. The income gain is three quarters of that of the base case. The results are thus quite robust to a wide range of labour supply elasticity assumptions.

Chart 2a: Sensitivity Tests (Total Impacts)







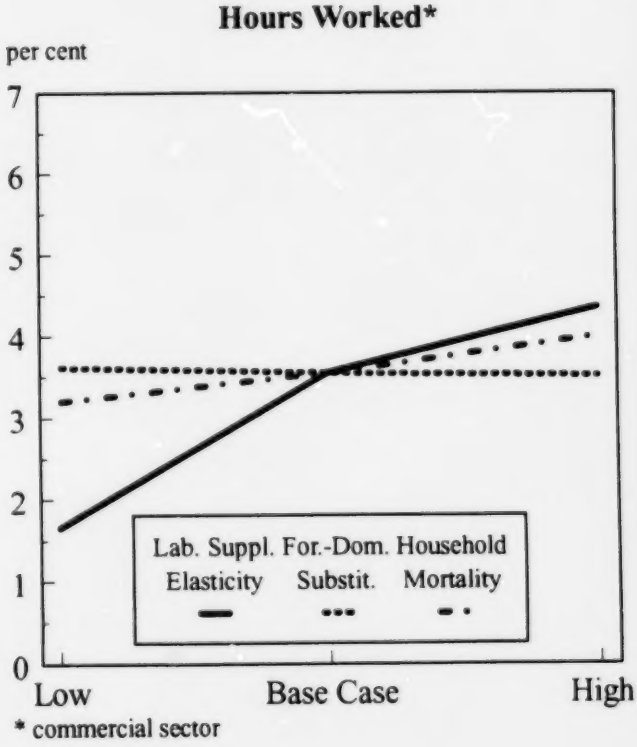
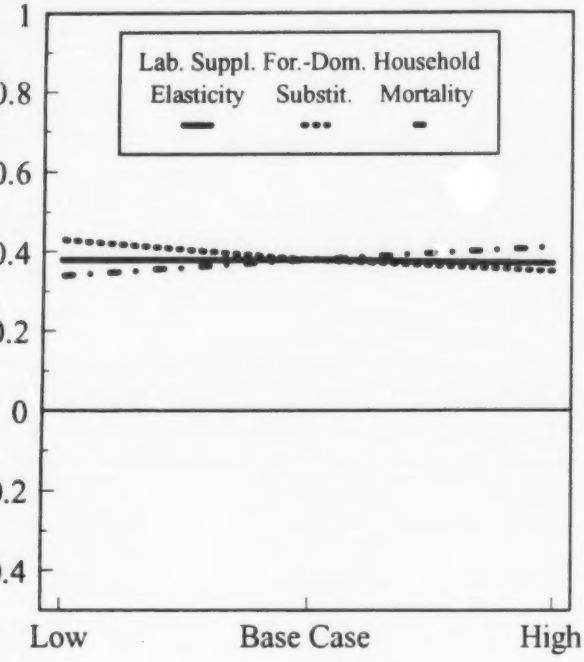


Chart 2b: Sensitivity Tests (Total Impacts)

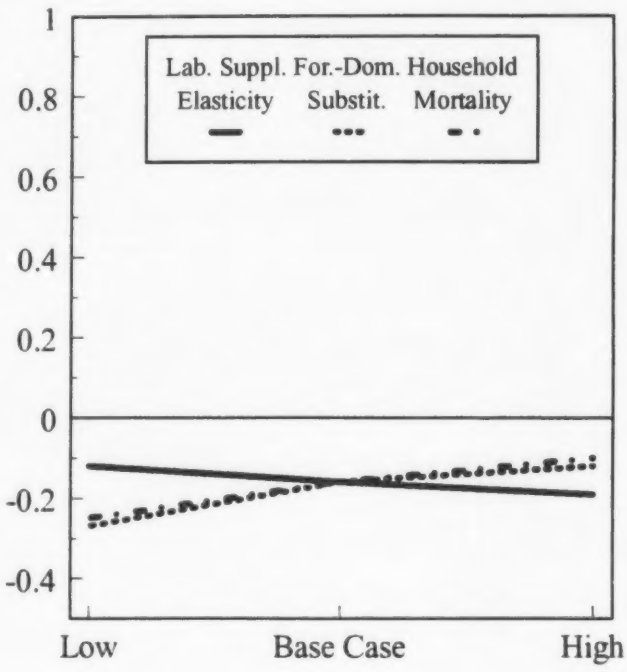
Gross Saving

er cent of GDP



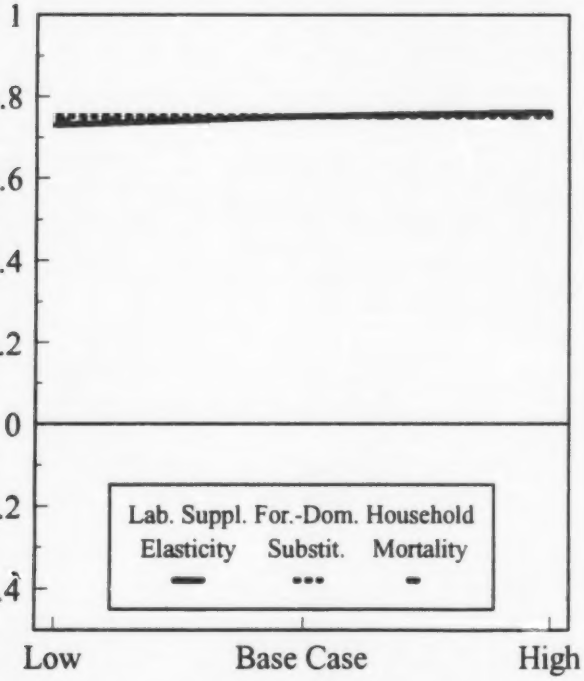
Private Domestic Saving

per cent of GDP



Public Domestic Saving

r cent of GDP



Net Foreign Saving

per cent of GDP

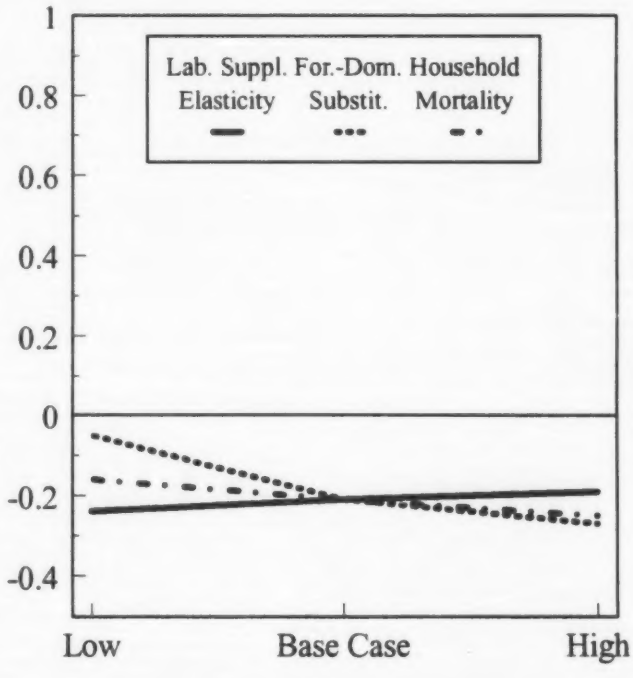


Table 3a: Long-Run Economic Impacts Given Alternative Labour Supply Elasticity Values

per cent shock minus control	Labour Supply Elasticity	Savings Channel	Tax Channel	Total
Gross Domestic Product	Low (0.25)	0.74	1.21	1.95
	Base Case (1.0)	0.70	2.30	2.99
	High (2.0)	0.56	2.86	3.42
Gross National Product	Low (0.25)	1.72	1.01	2.74
	Base Case (1.0)	1.43	2.28	3.71
	High (2.0)	1.11	2.97	4.07
Commercial Output	Low (0.25)	1.11	1.81	2.92
	Base Case (1.0)	1.04	3.44	4.49
	High (2.0)	0.84	4.29	5.13
Consumption	Low (0.25)	2.07	1.41	3.48
	Base Case (1.0)	1.77	3.31	5.08
	High (2.0)	1.39	4.33	5.71
Total Domestic Saving	Low (0.25)	5.01	2.05	7.06
	Base Case (1.0)	3.90	4.02	7.92
	High (2.0)	2.94	5.19	8.14
Capital Stock (Commercial)	Low (0.25)	2.40	2.46	4.87
	Base Case (1.0)	1.95	3.98	5.93
	High (2.0)	1.50	4.82	6.32
Hours Worked (Commercial)	Low (0.25)	0.27	1.38	1.65
	Base Case (1.0)	0.45	3.09	3.54
	High (2.0)	0.41	3.94	4.35

Table 3b: Long-Run Impacts on Components of Savings and Net Foreign Debt Given Alternative Labour Supply Elasticity Values

per cent of GDP; shock minus control	Labour Supply Elasticity	Savings Channel	Tax Channel	Total
Gross Saving	Low (0.25)	0.21	0.16	0.38
	Base Case (1.0)	0.16	0.22	0.38
	High (2.0)	0.12	0.25	0.37
Private Domestic Saving	Low (0.25)	-0.21	-0.08	-0.12
	Base Case (1.0)	-0.33	0.17	-0.16
	High (2.0)	-0.43	0.23	-0.19
Public Domestic Saving	Low (0.25)	0.72	0.02	0.73
	Base Case (1.0)	0.72	0.03	0.75
	High (2.0)	0.71	0.04	0.76
Net Foreign Saving	Low (0.25)	-0.30	0.06	-0.24
	Base Case (1.0)	-0.22	0.01	-0.21
	High (2.0)	-0.16	-0.03	-0.19
Net Foreign Debt	Low (0.25)	-10.20	2.11	-8.09
	Base Case (1.0)	-7.62	0.42	-7.19
	High (2.0)	-5.66	-0.86	-6.52

Tables 4a and 4b provide the results of resimulations conducted using alternative elasticities of substitution between foreign and domestic assets of 0.2 and 2.5, compared with the benchmark level of 1.43. As would be expected, the greater the degree of substitutability, the greater the SC impact on net foreign asset holdings and the smaller the SC impact on the capital stock. A higher substitutability means that a change in domestic saving has a smaller impact on domestic rates of return and, therefore, on investment and the capital stock. A higher portion of an increase in domestic saving will thus be channeled into a reduced reliance on foreign sources

Table 4a: Long-Run Economic Impacts Given Alternative Foreign-Domestic Asset Substitutability Values

per cent shock minus control	Foreign-Domestic Asset Substitutability	Savings Channel	Tax Channel	Total
Gross Domestic Product	Low (0.2)	0.82	2.35	3.17
	Base Case (1.4)	0.70	2.30	2.99
	High (2.5)	0.63	2.28	2.91
Gross National Product	Low (0.2)	1.10	2.36	3.45
	Base Case (1.4)	1.43	2.28	3.71
	High (2.5)	1.54	2.26	3.80
Commercial Output	Low (0.2)	1.23	3.53	4.76
	Base Case (1.4)	1.04	3.44	4.49
	High (2.5)	0.94	3.42	4.37
Consumption	Low (0.2)	1.36	3.42	4.78
	Base Case (1.4)	1.77	3.31	5.08
	High (2.5)	1.91	3.29	5.20
Total Domestic Saving	Low (0.2)	3.00	4.20	7.19
	Base Case (1.4)	3.90	4.02	7.92
	High (2.5)	4.21	3.98	8.19
Capital Stock (Commercial)	Low (0.2)	2.40	4.13	6.53
	Base Case (1.4)	1.95	3.98	5.93
	High (2.5)	1.73	3.94	5.67
Hours Worked (Commercial)	Low (0.2)	0.47	3.14	3.61
	Base Case (1.4)	0.45	3.09	3.54
	High (2.5)	0.43	3.08	3.51

of saving. The domestic saving impact itself increases as the elasticity rises owing to a smaller private saving offset stemming from the reduced rate of return decline. Although the SC GDP impact falls by about 0.2 percentage points as the elasticity of substitution is raised from 0.2 to

2.5, the GNP impact rises by 0.4 percentage points as a result of reduced net foreign debt servicing. The TC effects are essentially invariant with respect to the elasticity.

Table 4b: Long-Run Impacts on Components of Savings and Net Foreign Debt Given Alternative Foreign-Domestic Asset Substitutability Values

per cent of GDP; shock minus control	Foreign-Domestic Asset Substitutability	Savings Channel	Tax Channel	Total
Gross Saving	Low (0.2)	0.20	0.23	0.43
	Base Case (1.4)	0.16	0.22	0.38
	High (2.5)	0.14	0.21	0.35
Private Domestic Saving	Low (0.2)	-0.46	0.18	-0.27
	Base Case (1.4)	-0.33	0.17	-0.16
	High (2.5)	-0.29	0.17	-0.12
Public Domestic Saving	Low (0.2)	0.72	0.03	0.75
	Base Case (1.4)	0.72	0.03	0.75
	High (2.5)	0.71	0.03	0.75
Net Foreign Saving	Low (0.2)	-0.06	0.01	-0.05
	Base Case (1.4)	-0.22	0.01	-0.21
	High (2.5)	-0.29	0.01	-0.27
Net Foreign Debt	Low (0.2)	-1.96	0.28	-1.68
	Base Case (1.4)	-7.62	0.42	-7.19
	High (2.5)	-9.86	0.43	-9.44

Tables 5a and 5b show the long-run impacts given alternative household mortality rates. A zero household mortality rate implies absolute Ricardian Equivalence and, consequently, zero SC impacts. The SC impacts are, as would be expected, quite sensitive to the mortality rate. The larger TC effects, however, vary only slightly with the mortality rate, with the consequence that the overall results are again quite robust. A tripling of the mortality rate from 0.02 to 0.06 raises the long-run income gain from 3.2 per cent to 4.2 cent.

Table 5a: Long-Run Economic Impacts Given Alternative Household Mortality Rates

per cent shock minus control	Household Mortality Rate	Savings Channel	Tax Channel	Total
Gross Domestic Product	Low (0.02)	0.38	2.30	2.68
	Base Case (0.04)	0.70	2.30	2.99
	High (0.06)	1.03	2.30	3.33
Gross National Product	Low (0.02)	0.86	2.35	3.22
	Base Case (0.04)	1.43	2.28	3.71
	High (0.06)	1.93	2.24	4.17
Commercial Output	Low (0.02)	0.57	3.45	4.02
	Base Case (0.04)	1.04	3.44	4.49
	High (0.06)	1.54	3.45	4.99
Consumption	Low (0.02)	1.04	3.39	4.43
	Base Case (0.04)	1.77	3.31	5.08
	High (0.06)	2.45	3.28	5.73
Total Domestic Saving	Low (0.02)	2.48	4.33	6.80
	Base Case (0.04)	3.90	4.02	7.92
	High (0.06)	4.99	3.83	8.83
Capital Stock (Commercial)	Low (0.02)	1.19	4.10	5.30
	Base Case (0.04)	1.95	3.98	5.93
	High (0.06)	2.60	3.92	6.52
Hours Worked (Commercial)	Low (0.02)	0.16	3.03	3.19
	Base Case (0.04)	0.45	3.09	3.54
	High (0.06)	0.84	3.15	3.99

Table 5b: Long-Run Impacts on Components of Savings and Net Foreign Debt Given Alternative Household Mortality Rates

per cent of GDP; shock minus control	Household Mortality Rate	Savings Channel	Tax Channel	Total
Gross Saving	Low (0.02)	0.11	0.23	0.34
	Base Case (0.04)	0.16	0.22	0.38
	High (0.06)	0.20	0.21	0.41
Private Domestic Saving	Low (0.02)	-0.46	0.21	-0.25
	Base Case (0.04)	-0.33	0.17	-0.16
	High (0.06)	-0.24	0.15	-0.10
Public Domestic Saving	Low (0.02)	0.71	0.03	0.74
	Base Case (0.04)	0.72	0.03	0.75
	High (0.06)	0.72	0.03	0.75
Net Foreign Saving	Low (0.02)	-0.15	-0.01	-0.16
	Base Case (0.04)	-0.22	0.01	-0.21
	High (0.06)	-0.27	0.03	-0.25
Net Foreign Debt	Low (0.02)	-5.04	-0.33	-5.37
	Base Case (0.04)	-7.62	0.42	-7.19
	High (0.06)	-9.36	0.89	-8.46

5 Summary and Conclusions

This paper presents simulations of a dynamic computable general equilibrium model incorporating generational disconnectedness and distorting taxes to determine the long-run effects of debt reduction in Canada. We find that a debt-to-GDP ratio reduction of 25 percentage points raises real commercial sector output in the long run by 4.5 per cent, real GDP by 3 per cent, real GNP by 3.7 per cent and real consumption by 5.1 per cent.

These gains occur through two channels: first, because of increased domestic saving that raises the capital stock and lowers net foreign indebtedness; and, second, because of the

reduction in distorting taxes made possible by achieving a lower sustainable debt-to-GDP ratio that further boosts the capital stock and raises hours worked. The simulations have been conducted under the assumption that the savings in government debt servicing are applied fully to a uniform proportional reduction in all taxes, with expenditures unchanged.

The positive economic effects coming through reductions in taxes exceed those coming through increased national savings. The former would naturally be altered if taxes were lowered differently from the proportional assumption made in the analysis. Economic gains would be larger if the reductions were focused on the most distorting taxes. They would, however, be smaller if either expenditures were increased or less distorting taxes were lowered.

A series of sensitivity tests demonstrate that the results are quite robust to a wide range of assumptions regarding key parameters. The greatest sensitivity is exhibited in the relationship between the increase in commercial sector hours worked and the real wage elasticity of labour supply. Even in this case, however, economic gains are substantial.

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Appendix - The Dynamic CGE Model: Technical Derivations

A.1 Household Optimization Problems

At each instant t, a domestic risky asset-holding (type A) household born at time $\tau \leq t$ solves the problem:

$$(1)\quad \max E\int_{t}^{\infty} e^{-(\rho+p)(s-t)} \ln\left\{[c^{A}(\tau,s)]^{u_1}[\Omega(\tau,s)]^{1-u_1}\right\} ds$$

$$\text{s.t. } d\,a^{A}(\tau,s) = \{[i^{GD}(s)(1-\varphi(\tau,s)) + i^{CD}(s)\varphi(\tau,s)]b(\tau,s)\,a^{A}(\tau,s)$$

$$+ [\hat{e}^{F}(s)\theta(\tau,s) + \hat{e}^{D}(s)(1-\theta(\tau,s))][1-b(\tau,s)]\,a^{A}(\tau,s) + p\,a^{A}(\tau,s)$$

$$- c^{AD}(\tau,s) - P^{FR}c^{AF}(\tau,s)\}ds + \{[1-b(\tau,s)][\theta(\tau,s)\sigma^{\hat{E}F}(s) + (1-\theta(\tau,s))\sigma^{\hat{E}D}(s)]$$

$$+ \varphi(\tau,s)b(\tau,s)\sigma^{\hat{B}CD}(s)\}\,a^{A}(\tau,s)dz$$

$$(2)\quad \Omega(\tau,s) \equiv u_2\{[1-q(0)]^{1-u_3}[1-q(\tau,s)]^{u_3} + [q(0)]^{1-u_3}[q(\tau,s)]^{u_3}\}^{1/u_3}$$

$$(3)\quad q(\tau,s) \equiv 1 - [1-b(\tau,s)]\theta(\tau,s)$$

$$(4)\quad q(0) \equiv 1 - [1-b(0)]\theta(0)$$

$$(5)\quad c^{A}(\tau,s) \equiv [c^{AD}(\tau,s)]^{u_4}[c^{AF}(\tau,s)]^{1-u_4}\ ;\ 0 \leq u_1,\ u_4 \leq 1\ ;\ \rho > 0\ ;\ p \geq 0$$

where ρ is the rate of time preference, p is the household mortality rate, c^{Aj} is household consumption of good j, a^{A} is the stock of household financial assets, i^{j} is the real after-personal tax return on bonds of type j, P^{FR} is the real exchange rate, e^{D} is the domestic equity return, $\hat{e}^{F}$ is the after-tax exchange rate-adjusted foreign equity return; b, θ, φ and q are asset shares, $(\sigma^{\hat{E}D})^2$, $(\sigma^{\hat{E}F})^2$ and $(\sigma^{\hat{B}CD})^2$ are the variances of the after-tax returns on domestic equity, foreign equity and domestic corporate bonds, z is a Weiner process (hence the distributions are lognormal), the superscript G denotes government, C, corporate, D, domestic and F, foreign. The elasticity of substitution between foreign and domestic assets is $1/(1-u_3)$ and in the case of zero substitutability, q(0) is the portfolio share of domestic assets. The after-tax returns and variances have the explicit forms:

$$(6)\quad i^{GD}(t) \equiv r^{GD}(t)\left[1-\tau^{KBD}(t)\right] - \pi^{C}(t)$$

$$(7)\quad i^{CD}(t) \equiv r^{CD}(t)\left[1-\tau^{KBD}(t)\right] - \pi^{C}(t)$$

(8) $$\hat{e}^{F}(t) \equiv \left[1-\tau^{KE}(t)\right]e^{F}(t)+\left[1-\tau^{KGD}(t)\right]\left[\pi^{F}(t)-\dot{\varepsilon}(t)\right]-\pi^{C}(t)$$

(9) $$\hat{e}^{D}(t) \equiv \left[1-\tau^{KE}(t)\right]e^{D}(t)$$

(10) $$\left[\sigma^{\hat{E}F}(t)\right]^{2} = \left[1-\tau^{KE}(t)\right]^{2}\left[\sigma^{EF}(t)\right]^{2}$$

(11) $$\left[\sigma^{\hat{E}D}(t)\right]^{2} = \left[1-\tau^{KE}(t)\right]^{2}\left[\sigma^{ED}(t)\right]^{2}$$

(12) $$\left[\sigma^{B\hat{C}D}(t)\right]^{2} = \left[1-\tau^{KBD}(t)\right]^{2}\left[\sigma^{BCD}(t)\right]^{2}$$

where r^{j} is the nominal return on asset j, τ^{KBD} is the personal tax rate on interest income, τ^{KE} is the effective tax rate on equity income, ε is the value of the domestic currency expressed in units of the foreign currency and π^{C} is the rate of consumption price inflation.

At each instant t, a domestic non-risky asset-holding worker (type B) household born at time $\tau \le t$ solves the problem:

(13) $$\max E\int_{t}^{\infty} e^{-(\rho+p)(s-t)}\frac{\left\{[c^{B}(\tau,s)]^{u_5}[n(\tau,s)^{1-u_5}]\right\}^{\xi}}{\xi}ds$$

$$s.t.\, da^{B}(\tau,s) = \left\{\left[i^{GD}(s)+p\right]a^{B}(\tau,s)+[\bar{n}-n(\tau,s)]\frac{\hat{w}(s)}{\hat{P}(s)}-T(\tau,s)-c^{BD}(\tau,s)-P^{FR}(s)c^{BF}(\tau,s)\right\}ds$$

$$c^{B}(\tau,s) \equiv \left[c^{BD}(\tau,s)\right]^{u_4}\left[c^{BF}(\tau,s)\right]^{1-u_4};0 \le u_4,u_5 \le 1;\rho > 0;p \ge 0$$

where n is the consumption of leisure, $\bar{n}$ is the time endowment, $\hat{w}$ is the after-tax nominal wage, $\hat{P}$ is the after-tax consumption good price index and T is taxes net of transfers. The momentary elasticity of labour supply is $(1 - u_5)/u_5$ and the intertemporal elasticity of substitution is $1/(1 - \xi)$.

The optimal consumption solutions are given by:

(14) $$c^{A}(\tau,t) = (\rho+p)a^{A}(\tau,t)$$

(15) $$c^{kD}(\tau,t) = u_4 c^{k}(\tau,t)\left[P^{FR}(t)\right]^{1-u_4};k = A,B$$

(16) $$c^{kF}(\tau,t) = (1-u_4)c^{k}(\tau,t)\left[P^{FR}(t)\right]^{-u_4};k = A,B$$

(17) $$c^{B}(\tau,t) = m(t)u_5 W^{B}(\tau,t)$$

$$(18)\quad n(\tau,t)=m(t)(1-u_5)W^B(\tau,t)\frac{\hat{P}(t)}{\hat{w}(t)}$$

where:

$$(19)\quad \frac{1}{m(t)}=\int_t^{\infty}\exp\left\{\int_t^s\left(\frac{\xi}{1-\xi}\right)\left\{i^{GD}(u)+(1-u_5)\left[\pi^C(u)-\pi^W(u)\right]\right\}du-\left(\frac{\rho}{1-\xi}+p\right)s\right\}ds$$

and π^C and π^W are rates of consumption price and wage inflation, respectively.

The model is coded in discrete time. The discrete time analog of (15), for example, may be written in the recursive form:

$$(20)\quad \frac{1}{m(t)}=1+\frac{\left[(1+i^{GD}(t))\left(\frac{1+\pi^C(t+1)}{1+\pi^W(t+1)}\right)^{1-u_5}\right]^{\frac{\xi}{\xi-1}}}{m(t+1)\left[(1+\rho)^{\frac{1}{1-\xi}}(1+p)\right]^{s-t}}$$

Wealth is the sum of human and financial wealth:

$$(21)\quad \begin{aligned}W^B(\tau,t)&=\int_t^{\infty}-\dot{T}(\tau,t)\exp\left\{\int_t^s-\left[i^{GD}(u)+p\right]du\right\}ds+a^B(\tau,t)\\&+\frac{\hat{w}(t)}{\hat{P}(t)}\int_t^{\infty}\bar{n}\exp\left\{\int_t^s-\left[i^{GD}(u)+\pi^C(u)-\pi^W(u)+p\right]du\right\}ds\end{aligned}$$

The aggregate solutions are:

$$(22)\quad C^A(t)=(\rho+p)A^A(t)$$

$$(23)\quad C^{kD}(t)=u_4C^k(t)\left[P^{FR}(t)\right]^{1-u_4};k=A,B$$

$$(24)\quad C^{kF}(t)=(1-u_4)C^k(t)\left[P^{FR}(t)\right]^{-u_4};k=A,B$$

$$(25)\quad C^B(t)=m(t)u_5\tilde{W}^B(t)$$

$$(26)\quad N(t)=m(t)(1-u_5)\tilde{W}^B(t)\frac{\hat{P}(t)}{\hat{w}(t)}$$

where:

$$\tilde{W}^B(t) = \int_t^{\infty} -\tilde{T}(t)\exp\left\{\int_t^s -\left[i^{GD}(u) + p + g^N\right]du\right\}ds + A^B(t)$$

(27)

$$+\frac{\hat{w}(t)}{\hat{P}(t)}\int_t^{\infty}\overline{N}(t)\exp\left\{\int_t^s -\left[i^{GD}(u) + \pi^C(u) - \pi^W(u) + p + g^H\right]du\right\}ds$$

where upper case or bold symbols and "~" denote aggregates and g^H is the growth rate of households, so that p + g^H is the household birth rate.

The time endowment evolves according to:

$$\frac{d\overline{N}(t)}{\overline{N}(t)} = g^N dt \qquad (28)$$

where $g^N > 0$ is the population growth rate. Labour supply equals the time endowment minus consumption of leisure:

$$L^S(t) = \overline{N}(t) - N(t) \qquad (29)$$

Given zero asset return co-variances, the optimal portfolio shares of the domestic type-A households are given by:

$$\left(\frac{1-u_1}{u_1}\right)\frac{\rho}{\Omega(t)}\cdot\frac{\partial\Omega}{\partial b} + i^{GD}(t)[1-\varphi(t)] + i^{CD}(t)\varphi(t) - e^F(t)\theta(t) - e^D(t)[1-\theta(t)]$$

(30)

$$+[1-b(t)]\left\{[\theta(t)]^2\left[\sigma^{EF}(t)\right]^2 + [1-\theta(t)]^2\left[\sigma^{ED}(t)\right]^2\right\} + b(t)[\varphi(t)]^2\left[\sigma^{BCD}(t)\right]^2 = 0$$

$$\left(\frac{1-u_1}{u_1}\right)\frac{\rho}{\Omega(t)}\cdot\frac{\partial\Omega}{\partial\theta} + \left[e^F(t) - e^D(t)\right][1-b(t)] - [1-b(t)]^2\left\{\theta(t)\left[\sigma^{EF}(t)\right]^2 - [1-\theta(t)]\left[\sigma^{ED}(t)\right]^2\right\} = 0 \qquad (31)$$

$$i^{CD}(t) - i^{GD}(t) - b(t)\varphi(t)\left[\sigma^{BCD}(t)\right]^2 = 0 \qquad (32)$$

where:

$$(33)\quad \frac{1}{\Omega(t)}\cdot\frac{\partial\Omega}{\partial b}=\frac{\theta(t)\left[\left(\frac{q(0)}{q(t)}\right)^{1-\nu_3}-\left(\frac{1-q(0)}{1-q(t)}\right)^{1-\nu_3}\right]}{[1-q(0)]^{1-\nu_3}[1-q(t)]^{\nu_3}+[q(0)]^{1-\nu_3}[q(t)]^{\nu_3}}$$

$$(34)\quad \frac{1}{\Omega(t)}\cdot\frac{\partial\Omega}{\partial\theta}=\frac{-[1-b^F(t)]\left[\left(\frac{q(0)}{q(t)}\right)^{1-\nu_3}-\left(\frac{1-q(0)}{1-q(t)}\right)^{1-\nu_3}\right]}{[1-q(0)]^{1-\nu_3}[1-q(t)]^{\nu_3}+[q(0)]^{1-\nu_3}[q(t)]^{\nu_3}}$$

$$(35)\quad q(t)=1-[1-b(t)]\theta(t)$$

The stochastic portion of government capital tax revenue is assumed to be offset by the application of stochastic lump-sum capital taxes and transfers.

Foreign agents solve similar maximization problems. At each instant t, a domestic risky asset-holding (type A) household born at time $\tau \le t$ solves the problem:

$$(36)\quad \max E\int_t^\infty e^{-(\rho+p)(s-t)}\ln\left\{[c^{FA}(\tau,s)]^{\nu_1}[\Omega^F(\tau,s)\Omega^{EF}(\tau,s)\Omega^{BF}(\tau,s)]^{1-\nu_1}\right\}ds$$

$$\text{s.t. } da^{AF}(\tau,s) = \{[i^{GFF}(s)[1-\varphi^F(\tau,s)]+i^{CDF}(s)\varphi^F(\tau,s)]b^F(\tau,s)[1-\mu^F(\tau,s)]a^{AF}(\tau,s)$$

$$+[i^{GDF}(s)[1-\lambda^F(\tau,s)]+i^{CDF}(s)\lambda^F(\tau,s)]b^F(\tau,s)\mu^F(\tau,s)a^{AF}(\tau,s)$$

$$+[\hat{e}^{DF}(s)\theta^F(\tau,s)+\hat{e}^{FF}(s)[1-\theta^F(\tau,s)]][1-b^F(\tau,s)]a^{AF}(\tau,s)+pa^{AF}(\tau,s)$$

$$-c^{AFF}(\tau,s)-\frac{c^{ADF}(\tau,s)}{P^{FR}}\}ds+\{[1-b^F(\tau,s)][\theta^F(\tau,s)\sigma^{E\hat{D}F}(s)+[1-\theta^F(\tau,s)]\sigma^{E\hat{F}F}(s)]$$

$$+\varphi^F(\tau,s)[1-\mu^F(\tau,s)]b^F(\tau,s)\sigma^{B\hat{C}FF}(s)+\lambda^F(\tau,s)\mu^F(\tau,s)b^F(\tau,s)\sigma^{BC\hat{D}F}(s)\}a^{AF}(\tau,s)dz$$

$$(37)\quad \Omega^F(\tau,s)\equiv\nu_2\{[1-q(0)]^{1-\nu_3}[1-q(\tau,s)]^{\nu_3}+[q(0)]^{1-\nu_3}[q(\tau,s)]^{\nu_3}\}^{1/\nu_3}$$

$$(38)\quad \Omega^{EF}(\tau,s)\equiv\nu_2\{[1-\omega(0)]^{1-\nu_3}[1-\omega(\tau,s)]^{\nu_3}+[\omega(0)]^{1-\nu_3}[\omega(\tau,s)]^{\nu_3}\}^{1/\nu_3}$$

$$(39)\quad \Omega^{BF}(\tau,s)\equiv\nu_2\{[1-\lambda(0)]^{1-\nu_3}[1-\lambda(\tau,s)]^{\nu_3}+[\lambda(0)]^{1-\nu_3}[\lambda(\tau,s)]^{\nu_3}\}^{1/\nu_3}$$

(40) $q^F(\tau,s) \equiv 1 - \mu^F(\tau,s)b^F(\tau,s) - [1 - b^F(\tau,s)]\theta^F(\tau,s)$

(41) $q^F(0) \equiv 1 - \mu^F(0)b^F(0) - [1 - b^F(0)]\theta^F(0)$

(42) $$\omega^F(\tau,s) = \frac{\mu^F(\tau,s)b^F(\tau,s)}{\mu^F(\tau,s)b^F(\tau,s) + \left[1 - b^F(\tau,s)\right]\theta^F(\tau,s)}$$

(43) $$\omega^F(0) = \frac{\mu^F(0)b^F(0)}{\mu^F(0)b^F(0) + \left[1 - b^F(0)\right]\theta^F(0)}$$

(44) $c^{FA}(\tau,s) \equiv [c^{ADF}(\tau,s)]^{\nu_4}\,[c^{AFF}(\tau,s)]^{1-\nu_4}$; $0 \le \nu_1,\ \nu_4 \le 1$; $\rho > 0$; $p \ge 0$

where ω^F is the share of the foreign A-agent's domestic assets held as domestic bonds, λ^F is the share of the foreign A-agent's domestic bond portfolio held as domestic corporate bonds, μ^F is the share of the foreign A-agent's bond portfolio held as domestic bonds, and:

(45) $i^{GFF}(t) \equiv r^{GF}(t)\left[1 - \tau^{KBF}(t)\right] - \pi^F(t)$

(46) $i^{CFF}(t) \equiv r^{CF}(t)\left[1 - \tau^{KBF}(t)\right] - \pi^F(t)$

(47) $i^{GDF}(t) \equiv r^{GD}(t)\left[1 - \tau^{KBF}(t)\right] + \left[1 - \tau^{KGF}(t)\right]\dot{\varepsilon}(t) - \pi^F(t)$

(48) $i^{CDF}(t) \equiv r^{CD}(t)\left[1 - \tau^{KBF}(t)\right] + \left[1 - \tau^{KGF}(t)\right]\dot{\varepsilon}(t) - \pi^F(t)$

(49) $\hat{e}^{DF}(t) \equiv \left[1 - \tau^{KEF}(t)\right]e^D(t) + \left[1 - \tau^{KGF}(t)\right]\left[\pi^D(t) + \dot{\varepsilon}(t)\right] - \pi^F(t)$

(50) $\hat{e}^{FF}(t) \equiv \left[1 - \tau^{KEF}(t)\right]e^F(t)$

(51) $\left[\sigma^{E\hat{F}F}(t)\right]^2 = \left[1 - \tau^{KEF}(t)\right]^2\left[\sigma^{EF}(t)\right]^2$

(52) $\left[\sigma^{E\hat{D}F}(t)\right]^2 = \left[1 - \tau^{KEF}(t)\right]^2\left[\sigma^{ED}(t)\right]^2$

(53) $\left[\sigma^{BC\hat{D}F}(t)\right]^2 = \left[1 - \tau^{KBDF}(t)\right]^2\left[\sigma^{BCD}(t)\right]^2$

(54) $\left[\sigma^{BC\hat{F}F}(t)\right]^2 = \left[1 - \tau^{KBDF}(t)\right]^2\left[\sigma^{BCF}(t)\right]^2$

The optimal portfolio shares of the foreign A-agent are given by simultaneous solution of (55)-(72):

$$\rho\left(\frac{1-\nu_1}{\nu_1}\right)\left[\frac{1}{\Omega^F(t)}\cdot\frac{\partial\Omega^F}{\partial b^F}+\frac{1}{\Omega^{EF}(t)}\cdot\frac{\partial\Omega^{EF}}{\partial b^F}\right]$$
$$+\left\{i^{GFF}(t)\left[1-\varphi^F(t)\right]+i^{CFF}(t)\varphi^F(t)\right\}\left[1-\mu^F(t)\right]$$
$$+\left\{i^{CDF}(t)\lambda^F(t)+i^{GDF}(t)\left[1-\lambda^F(t)\right]\right\}\mu^F(t)-\hat{e}^{DF}(t)\theta^F(t)-\hat{e}^{FF}(t)\left[1-\theta^F(t)\right]$$
$$+\left[1-b^F(t)\right]\left\{\left[\theta^F(t)\right]^2\left[\sigma^{E\hat{D}F}(t)\right]^2+\left[1-\theta^F(t)\right]^2\left[\sigma^{EFF}(t)\right]^2\right\}+b(t)\left[\varphi(t)\right]^2\left[\sigma^{BCD}(t)\right]^2$$
(55) $$+b^F(t)\left\{\left[1-\mu^F(t)\right]^2\left[\varphi^F(t)\right]^2\left[\sigma^{B\hat{C}F}(t)\right]^2+\left[\mu^F(t)\right]^2\left[\lambda^F(t)\right]^2\left[\sigma^{B\hat{C}D}(t)\right]^2\right\}=0$$

(56) $$\rho\left(\frac{1-\nu_1}{\nu_1}\right)\left[\frac{1}{\Omega^F(t)}\cdot\frac{\partial\Omega^F}{\partial\theta^F}+\frac{1}{\Omega^{EF}(t)}\cdot\frac{\partial\Omega^{EF}}{\partial\theta^F}\right]$$
$$+\left[1-b^F(t)\right]\left[\hat{e}^{DF}(t)-\hat{e}^{FF}(t)\right]+\left[1-b^F(t)\right]^2\left\{\theta^F(t)\left[\sigma^{E\hat{D}F}(t)\right]^2-\left[1-\theta^F(t)\right]\left[\sigma^{E\hat{F}F}(t)\right]^2\right\}=0$$

$$\rho\left(\frac{1-\nu_1}{\nu_1}\right)\left[\frac{1}{\Omega^F(t)}\cdot\frac{\partial\Omega^F}{\partial\mu^F}+\frac{1}{\Omega^{EF}(t)}\cdot\frac{\partial\Omega^{EF}}{\partial\mu^F}\right]$$
(57) $$+\left\{i^{GFF}(t)\left[1-\varphi^F(t)\right]+i^{CFF}(t)\varphi^F(t)\right\}b^F(t)+\left\{i^{CDF}(t)\lambda^F(t)+i^{GDF}(t)\left[1-\lambda^F(t)\right]\right\}b^F(t)$$
$$+\left[b^F(t)\right]^2\left\{\left[1-\mu^F(t)\right]\left[\varphi^F(t)\right]^2\left[\sigma^{B\hat{C}F}(t)\right]^2+\left[\mu^F(t)\right]^2\left[\sigma^{B\hat{C}D}(t)\right]^2\right\}=0$$

(58) $$i^{CFF}(t)-i^{GFF}(t)-b^F(t)\varphi^F(t)\left[\sigma^{B\hat{C}F}(t)\right]^2=0$$

(59) $$\rho\left(\frac{1-\nu_1}{\nu_1}\right)\left[\frac{1}{\Omega^{BF}(t)}\cdot\frac{\partial\Omega^{BF}}{\partial\lambda^F}\right]+\left\{i^{CDF}(t)-i^{GDF}(t)\left[1-\lambda^F(t)\right]\right\}b^F(t)\mu^F(t)$$
$$+\left[\mu^F(t)\right]^2\lambda^F(t)\left[b^F(t)\right]^2\left[\sigma^{B\hat{C}D}(t)\right]^2=0$$

(60) $$q^F(t)=1-\mu^F(t)b^F(t)-\left[1-b^F(t)\right]\beta^F(t)$$

(61) $$q^F(t)=\frac{\mu^F(t)b^F(t)}{\mu^F(t)b^F(t)+\left[1-b^F(t)\right]\beta^F(t)}$$

(62) $$\frac{1}{\Omega^F(t)}\cdot\frac{\partial\Omega^F}{\partial b^F}=\frac{\left[\theta^F(t)-\mu^F(t)\right]\left[\left(\frac{q^F(0)}{q^F(t)}\right)^{1-\nu_3}-\left(\frac{1-q^F(0)}{1-q^F(t)}\right)^{1-\nu_3}\right]}{\left[1-q^F(0)\right]^{1-\nu_3}\left[1-q^F(t)\right]^{\nu_3}+\left[q^F(0)\right]^{1-\nu_3}\left[q^F(t)\right]^{\nu_3}}$$

(63) $$\frac{1}{\Omega^F(t)}\cdot\frac{\partial\Omega^F}{\partial\theta^F}=\frac{-\left[1-b^F(t)\right]\left[\left(\frac{q^F(0)}{q^F(t)}\right)^{1-\nu_3}-\left(\frac{1-q^F(0)}{1-q^F(t)}\right)^{1-\nu_3}\right]}{\left[1-q^F(0)\right]^{1-\nu_3}\left[1-q^F(t)\right]^{\nu_3}+\left[q^F(0)\right]^{1-\nu_3}\left[q^F(t)\right]^{\nu_3}}$$

(64) $$\frac{1}{\Omega^F(t)}\cdot\frac{\partial\Omega^F}{\partial\mu^F}=\frac{-b^F(t)\left[\left(\frac{q^F(0)}{q^F(t)}\right)^{1-\nu_3}-\left(\frac{1-q^F(0)}{1-q^F(t)}\right)^{1-\nu_3}\right]}{\left[1-q^F(0)\right]^{1-\nu_3}\left[1-q^F(t)\right]^{\nu_3}+\left[q^F(0)\right]^{1-\nu_3}\left[q^F(t)\right]^{\nu_3}}$$

(65) $$\frac{1}{\Omega^{EF}(t)}\cdot\frac{\partial\Omega^{EF}}{\partial\omega^F}=\frac{\left[\left(\frac{\omega^F(0)}{\omega^F(t)}\right)^{1-\nu_3}-\left(\frac{1-\omega^F(0)}{1-\omega^F(t)}\right)^{1-\nu_3}\right]}{\left[1-\omega^F(0)\right]^{1-\nu_3}\left[1-\omega^F(t)\right]^{\nu_3}+\left[\omega^F(0)\right]^{1-\nu_3}\left[\omega^F(t)\right]^{\nu_3}}$$

(66) $$\frac{1}{\Omega^{EF}(t)}\cdot\frac{\partial\Omega^{EF}}{\partial b^F}=\frac{1}{\Omega^{EF}(t)}\cdot\frac{\partial\Omega^{EF}}{\partial\omega^F}\frac{\partial\omega^F}{\partial b^F}$$

(67) $$\frac{1}{\Omega^{EF}(t)}\cdot\frac{\partial\Omega^{EF}}{\partial\theta^F}=\frac{1}{\Omega^{EF}(t)}\cdot\frac{\partial\Omega^{EF}}{\partial\omega^F}\frac{\partial\omega^F}{\partial\theta^F}$$

(68) $$\frac{1}{\Omega^{EF}(t)}\cdot\frac{\partial\Omega^{EF}}{\partial\mu^F}=\frac{1}{\Omega^{EF}(t)}\cdot\frac{\partial\Omega^{EF}}{\partial\omega^F}\frac{\partial\omega^F}{\partial\mu^F}$$

(69) $$\frac{1}{\Omega^{BF}(t)}\cdot\frac{\partial\Omega^{BF}}{\partial\lambda^F}=\frac{\left[\left(\frac{\lambda^F(0)}{\lambda^F(t)}\right)^{1-\nu_3}-\left(\frac{1-\lambda^F(0)}{1-\lambda^F(t)}\right)^{1-\nu_3}\right]}{\left[1-\lambda^F(0)\right]^{1-\nu_3}\left[1-\lambda^F(t)\right]^{\nu_3}+\left[\lambda^F(0)\right]^{1-\nu_3}\left[\lambda^F(t)\right]^{\nu_3}}$$

(70) $$\frac{\partial\omega^F}{\partial b^F}=\frac{\mu^F(t)\left\{\mu^F(t)b^F(t)+\left[1-b^F(t)\right]\theta^F(t)\right\}+\left[\theta^F(t)-\mu^F(t)\right]\mu^F(t)b^F(t)}{\left\{\mu^F(t)b^F(t)+\left[1-b^F(t)\right]\theta^F(t)\right\}^2}$$

(71) $$\frac{\partial\omega^F}{\partial\theta^F}=\frac{\mu^F(t)\left[1-b^F(t)\right]}{\left\{\mu^F(t)b^F(t)+\left[1-b^F(t)\right]\theta^F(t)\right\}^2}$$

$$(72)\quad \frac{\partial \omega^F}{\partial \mu^F} = \frac{b^F(t)\left\{\mu^F(t)b^F(t) + \left[1-b^F(t)\right]\beta^F(t)\right\} - \mu^F(t)\left[b^F(t)\right]^2}{\left\{\mu^F(t)b^F(t) + \left[1-b^F(t)\right]\beta^F(t)\right\}^2}$$

Exports are given by:

$$(73)\quad X(t) = (1-\nu_4)C^F(t)\left[P^{FR}(t)\right]^{\nu_4}$$

where C^F is total foreign agent consumption.

The net foreign asset position has the form:

$$(74)\quad W^{KN}(t) = \left[1-q(t)\right]A^A(t) - \left[1-q^F(t)\right]A^{AF}(t)P^{FR}(t)$$

The current account is given by:

$$(75)\quad \kappa^{CA}(t) = X(t) - C^F(t) - \psi I(t) + \kappa^{NS}(t)$$

where ψ is the portion of domestic investment imported from abroad and κ^{NS} is the net foreign servicing account.

The capital account equals the change in the net foreign asset position:

$$(76)\quad \kappa^{KA} = \Delta W^{KN}(t)$$

A.2 Firm Behaviour

Domestic representative firms invest in order to maximize the market value of equity, which is the discounted value of after-tax dividends net of new share issues:

$$(77)\quad S^D(t) = \int_t^{\infty}\left(\frac{1-\tau^{KDD}(t)}{1-\tau^{KGD}(t)}\right)D^D(s)\exp\left\{\int_t^s -\left(\frac{e^D(u)}{1-\tau^{KGD}(u)}\right)du\right\}ds$$

where D^D is dividends paid, and τ^{KGD} and τ^{KDD} are the capital gains and dividend tax rates, respectively. Firms face capital installation costs that are quadratic in investment intensity:

$$(78)\quad \varphi(t) = \begin{cases} \frac{\beta}{2}\left(\frac{I(t)}{K(t)} - \Theta\right)^2 \frac{K(t)}{I(t)}; \frac{I(t)}{K(t)} \geq \Theta \geq 0; \beta > 0 \\ 0; \frac{I(t)}{K(t)} < \Theta \end{cases}$$

where I is investment and the capital stock, K, evolves according to:

$$(79) \quad \frac{dK(t)}{dt} = I(t) - \delta K(t) \; ; \; 0 < \delta < 1 \qquad (31)$$

where δ is the depreciation rate. The production technology is given by:

$$(80) \quad Y(t) = q(t)[L(t)]^{\alpha} [K(t)]^{1-\alpha} \; ; \; q(t) > 0; \; 0 < \alpha < 1 \qquad (32)$$

$$(81) \quad \frac{dq(t)}{q(t)} = g^q dt; g^q > 0 \qquad (33)$$

where q is total factor productivity and L is hours worked. The approach to the solution of the investment problem taken here follows that of Summers (1981, 1985), which synthesizes the *q* theory of investment developed by Tobin (1969) and the adjustment cost framework of Lucas (1967). The optimal investment solution is:

$$(82) \quad \frac{I(t)}{K(t)} = \Theta + \frac{Q(t)}{\beta} \qquad (34)$$

where Q is the percentage deviation of the tax-adjusted value of the firm from the replacement price of the capital stock:

$$(83) \quad Q(t) = \frac{S^D(t) - \left[1 - \tau^{KDD}(t)\right] D^D(t) + B^{CD}(t) - Z(t)}{P^{KR}(t) K(t)} - 1 + \Gamma(t) + z(t)$$

where Z and z are the present values of capital consumption allowances on existing and new capital respectively. These have the discrete time forms:

$$(84) \quad Z(t) = \left\{\left[1 - \delta^T(t)\right] z(t) + \tau^{KC}(t) \delta^T(t)\right\} K^T(t-1)$$

$$(85) \quad z(t) = \frac{\tau^{KC}(t)\delta^T(t) + z(t+1)}{1 + \dfrac{\hat{e}^D(t) + \tau^{KGD}(t)\pi^D(t)}{1 - \tau^{KGD}(t)}}$$

where τ^{KC} is the corporate income tax rate, $\hat{e}^D$ is the after-domestic personal tax return on domestic equity and π^D is the domestic good inflation rate. The nominal and real capital consumption allowance bases are given by:

$$(86) \quad \tilde{K}^T(t) = \left[1 - \delta^T(t)\right]\left[\tilde{K}^T(t-1) + P^D(t) I(t-1)\right]$$

(87) $$K^T(t) = \frac{\tilde{K}^T(t)}{P^D(t)} z$$

Gross capital income is given by:

(88) $Y^K(t) = (1-\alpha)Y(t)$

Pre-corporate tax earnings equal net capital income minus real debt servicing payments:

(89) $$\hat{E}(t) = Y^K(t) - \delta K(t-1) - \left(\frac{1+r^{CD}(t-1)}{1+\pi^D(t-1)}\right) B^{CD}(t-1)$$

After-corporate tax earnings equal pre-corporate tax earnings minus corporate tax paid on a base of capital income net of CCA and nominal interest servicing, plus the inflation component of IVA:

(90) $$E(t) = \hat{E}(t) - \tau^{KC}(t)\left[Y^K(t) - \left(\frac{r^{CD}(t-1)}{1+\pi^D(t-1)}\right) B^{CD}(t-1)\right] + \tau^{KC}(t)\delta^T(t)K^T(t-1)$$
$$-\tau^{KC}(t)\pi(t)v\left[vY(t) + (1-v)Y(t-1)\right]$$

where v is the average inventory holding period. Earnings are either retained or paid out as dividends:

(91) $E(t) = R(t) + D(t)$

Investment is financed out of retained earnings and new debt issuance:

(92) $[1 - \Gamma(t) + \phi(t)]I(t) = R(t) + \Delta B^{CD}(t)$

Labour demand, L^D, is determined by equating the real producer wage with the marginal product of labour:

(93) $$\frac{w(t)}{P^D(t)} = \alpha q(t)\left[\frac{K(t)}{L^D(t)}\right]^{1-\alpha}$$

A.3 The Government

The government collects taxes net of transfers, T, absorbs resources, G, and supplies a public good, G^Y. For simplicity, we assume:

(94) $G^Y = G$

The government faces the debt accumulation condition:

$$(95) \quad \frac{dB^{GD}(t)}{dt} = G(t) - T(t) + i^{GD}(t)B(t)$$

This, combined with the No-Ponzi Game condition:

$$(96) \quad \lim_{t\to\infty} B^{GD}(\bar{t})\exp\{\int_{t}^{\bar{t}} -i^{GD}(s)ds\} = 0$$

implies:

$$(97) \quad B^{GD}(t) = \int_{t}^{\infty}[G(s) - T(s)]\exp\{\int_{t}^{s} -i^{GD}(u)du\}ds$$

The present value of future primary balances is:

$$(98) \quad W^{PR}(t) = \int_{t}^{\infty}[G(s) - T(s)]\exp\{\int_{t}^{s} -[i^{GD}(u) + p + g^{H}]du\}ds$$

and the net contribution of government debt to wealth is $B^{GD} - W^{PR}$.

A.4 General Equilibrium

Equations (99) to (104) combined with the balance of payments equilibrium condition (105) determine asset prices and the exchange rate. By Walras' law, one equation is redundant.

$$(99) \quad S^{D}(t) = [1 - b(t)][1 - \theta(t)]A^{A}(t) + [1 - b^{F}(t)]\theta^{F}(t)A^{AF}(t)P^{FR}(t)$$

$$(100) \quad B^{CD}(t) = b(t)\varphi(t)A^{A}(t) + \mu^{F}(t)\lambda^{F}(t)b^{F}(t)A^{AF}(t)P^{FR}(t)$$

$$(101) \quad B^{GD}(t) = b(t)[1 - \varphi(t)]A^{A}(t) + B^{GDB}(t) + \mu^{F}(t)[1 - \lambda^{F}(t)]b^{F}(t)A^{AF}(t)P^{FR}(t)$$

$$(102) \quad S^{F}(t)P^{FR}(t) = [1 - b(t)]\theta(t)A^{A}(t) + [1 - b^{F}(t)][1 - \theta^{F}(t)]A^{AF}(t)P^{FR}(t)$$

$$(103) \quad B^{CF}(t) = [1 - \mu^{F}(t)]\varphi^{F}(t)b^{F}(t)A^{AF}(t)$$

$$(104) \quad B^{GF}(t) = b^{F}(t)[1 - \mu^{F}(t)][1 - \varphi^{F}(t)]A^{AF}(t) + B^{GFB}(t)$$

$$(105) \quad \kappa^{CA}(t) + \kappa^{KA}(t) = 0$$

Equations (106) and (107) are the goods and labour market-clearing conditions, respectively:

(106) $q(t)[L(t)]^{\alpha}[K(t)]^{1-\alpha} + G^{Y}(t) = C^{D} + I^{D} + G^{A}(t) + X(t)$

(107) $L^{S}(t) = L^{D}(t)$

Table A.1 Calibration Values of Key Parameters

π^{C}	0.040	π^{W}	0.025	g^{H}	0.010	g^{N}	0.010
g^{q}	0.006	α	0.604	δ	0.070	ξ	-1.000
ρ	0.060	β	10.000	Θ	0.097	u_1	0.500
u_2	1.000	u_3	0.300	u_4	0.613	u_5	0.500
$(\sigma^{BCD})^2$	0.020	$(\sigma^{EQ})^2$	0.084	q(0)	0.976	$q^{F}(0)$	0.991
u_3	-4.000*	u_3	0.600**	q(0)	0.892*	q(0)	0.994**
$q^{F}(0)$	0.968*	$q^{F}(0)$	0.997**	A^{A}/A^{AF}	0.100	W^{B}/W^{BF}	0.100
p	0.040	u_5	0.80***	u_5	0.333†		

* foreign-domestic asset substitutability = 0.2
** foreign-domestic asset substitutability = 2.5
*** labour supply elasticity = 0.25
† labour supply elasticity = 2.0

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